Inventory Management

Chapter 12

Introduction

Inventory: Store of goods.

The inventory models described in this chapter relate primarily to what are referred to as **independent-demand** items, that is, items that are ready to be sold or used. Aggregate Planning describes models that are used for **dependent-demand** items, which are components of finished products, rather than the finished products themselves.

The Nature and Importance of Inventories

The different kinds of inventories include the following:

- Raw materials and purchased parts.
- Partially completed goods, called *work-in-process (WIP)*.
- Finished-goods inventories (manufacturing firms) or merchandise (retail stores).
- Replacement parts, tools, and supplies.
- Goods-in-transit to warehouses or customers (pipeline inventory).

Return on investment (ROI)

Functions of Inventory

- **To meet anticipated customer demand.** A customer can be a person who walks in off the street to buy a new stereo system, a mechanic who requests a tool at a tool crib, or a manufacturing operation. These inventories are referred to as *anticipation stocks* because they are held to satisfy expected (i.e., *average*) demand.

- **To smooth production requirements.** Firms that experience seasonal patterns in demand often build up inventories during preseason periods to meet overly high requirements during seasonal periods. These inventories are aptly named *seasonal inventories.*

- **To decouple operations.** companies have taken a closer look at buffer inventories, recognizing the cost and space they require, and realizing that finding and eliminating sources of disruptions can greatly decrease the need for decoupling operations.

- **To protect against stockouts.** The risk of shortages can be reduced by holding *safety stocks,* which are stocks in excess of average demand to compensate for *variabilities* in demand and lead time.

- **To take advantage of order cycles.** Inventory storage enables a firm to buy and produce in *economic lot sizes* without having to try to match purchases or production with demand requirements in the short run. This results in *periodic* orders, or order *cycles.* The resulting stock is known as *cycle stock.* Order cycles are not always based on economic lot sizes. In some instances, it is practical or economical to group orders and/or to order at fixed intervals.

- **To hedge against price increases.** Occasionally a firm will suspect that a substantial price increase is about to occur and purchase larger-than-normal amounts to beat the increase. The ability to store extra goods also allows a firm to take advantage of price discounts for larger orders.

- **To permit operations.** Little's Law can be useful in quantifying pipeline inventory. It states that the average amount of inventory in a system is equal to the product of the average rate at which inventory units leave the system (i.e., the average demand rate) and the average time a unit is in the system. Thus, if a unit is in the system for an average of 10 days, and the demand rate is 5 units per day, the average inventory is 50 units: 5 units/day × 10 days = 50 units.

- **To take advantage of quantity discounts.** Suppliers may give discounts on large orders.

Objectives of Inventory Control

Inventory Management

- the level of customer service
- the costs of ordering and carrying inventories

Two fundamental decisions: the *timing* and *size* of orders (i.e., when to order and how much to order).

Measures

1. **Inventory turnover**: the ratio of annual cost of goods sold to average inventory investment.
2. **Days of inventory on hand**

Requirements for Efficient Inventory Management

Management has two basic functions concerning inventory. One is **to establish a system of keeping track of items in inventory**, and the other is **to make decisions about how much and when to order**.

To be effective;

- A system to **keep track of the inventory** on hand and on order.
- A reliable **forecast of demand** that includes an indication of possible *forecast error*.
- Knowledge of **lead times** and **lead time variability**.
- Reasonable estimates of inventory **holding costs, ordering costs, and shortage costs**.
- A **classification system** for inventory items.

Inventory Counting Systems

Periodic system

Advantage: Orders for many items occur at the same time.

Disadvantages: Lack of control between reviews, need to protect against shortages between review periods by carrying extra stock.

Perpetual system (Continual system)

Advantages: Control provided by the continuous monitoring of inventory withdrawals, fixed-order quantity (management can determine an optimal order quantity).

Disadvantage: Added cost of record keeping.

Two-bin system

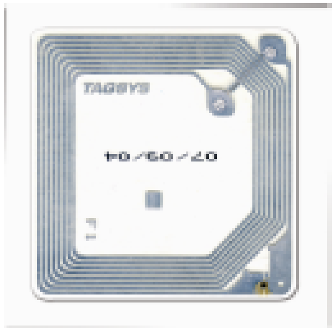
Advantage: There is no need to record each withdrawal from inventory.

Disadvantage: Reorder card may not be turned in for a variety of reasons.

Perpetual systems can be either *batch* or *online*.

Universal product code (UPC)

Radio frequency identification (RFID) tags



Demand Forecasts and Lead-Time Information

Point-of-sale (POS) systems

Inventory Costs

Holding (carrying) Costs: Costs include interest, insurance, taxes (in some states), depreciation, obsolescence, deterioration, spoilage, pilferage, breakage, and warehousing costs (heat, light, rent, security). They also include opportunity costs associated with having funds that could be used elsewhere tied up in inventory.

Transaction (ordering) Costs: Holding costs are stated in either of two ways: as a percentage of unit price or as a dollar amount per unit. When a firm produces its own inventory instead of ordering it from a supplier, the costs of machine setup (e.g., preparing equipment for the job by adjusting the machine, changing cutting tools) are analogous to ordering costs; that is, they are expressed as a fixed charge per production run, regardless of the size of the run.

Shortage Costs: These costs can include the opportunity cost of not making a sale, loss of customer goodwill, late charges, and similar costs. Furthermore, if the shortage occurs in an item carried for internal use (e.g., to supply an assembly line), the cost of lost production or downtime is considered a shortage cost.

Classification System

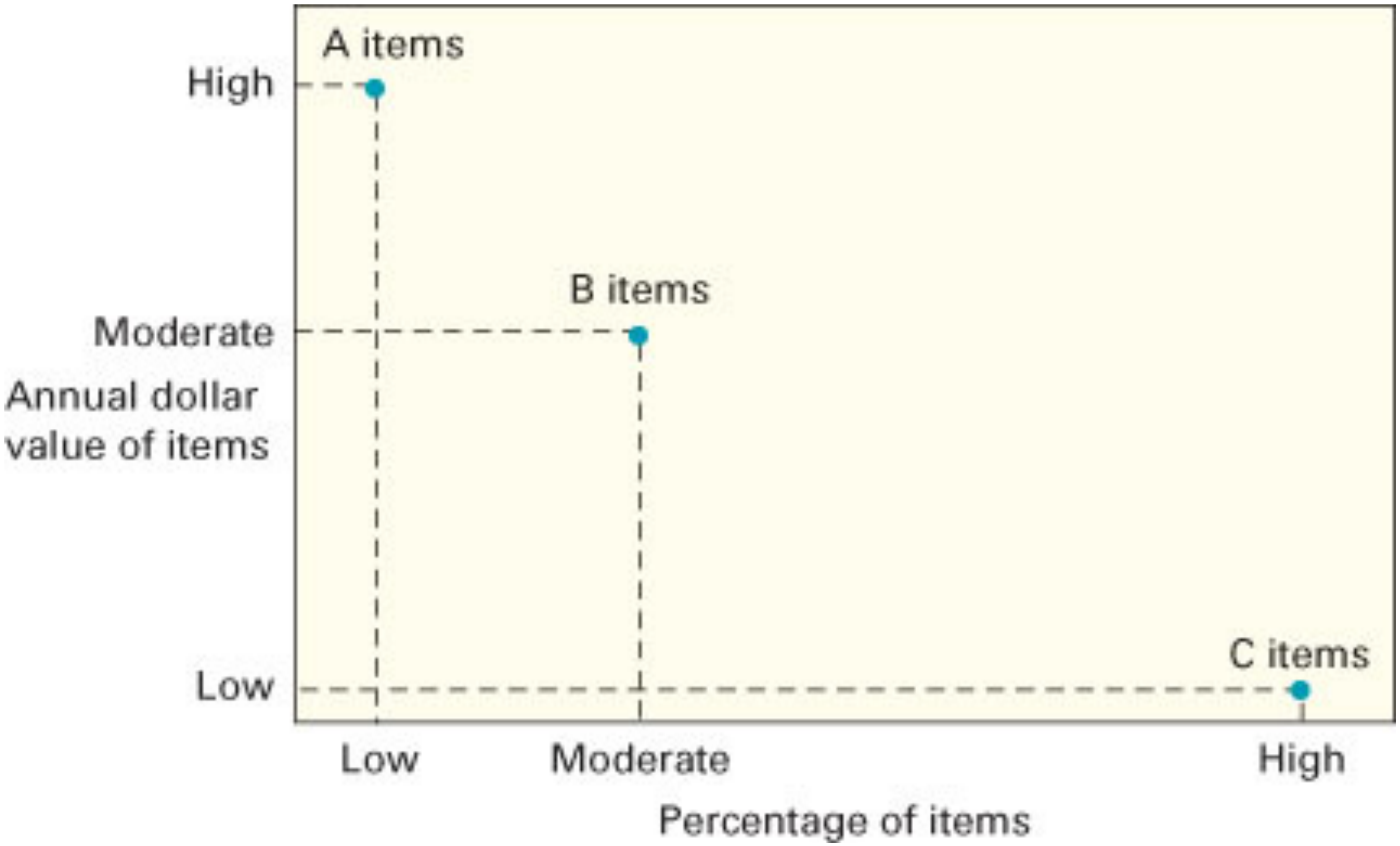
The **A-B-C approach** classifies inventory items according to some measure of importance, usually annual dollar value (i.e., dollar value per unit multiplied by annual usage rate), and then allocates control efforts accordingly. Typically, three classes of items are used: A (very important), B (moderately important), and C (least important).

Example

The annual dollar value of 12 items has been calculated according to annual demand and unit cost. The annual dollar values were then arrayed from highest to lowest to simplify classification of items.

Item Number	Annual Demand	×	Unit Cost	=	Annual Dollar Value	Classification
8	1,000		$4,000		$ 4,000,000	A
5	3,900		700		2,730,000	A
3	1,900		500		950,000	B
6	1,000		915		915,000	B
1	2,500		330		825,000	B
4	1,500		100		150,000	C
12	400		300		120,000	C
11	500		200		100,000	C
9	8,000		10		80,000	C
2	1,000		70		70,000	C
7	200		210		42,000	C
10	9,000		2		18,000	C
					10,000,000	

A typical A-B-C breakdown in relative annual dollar value of items and number of items by category



Cycle Counting

The key questions concerning cycle counting for management are

- How much accuracy is needed?
- When should cycle counting be performed?
- Who should do it?

APICS (The Association of Operations Management) recommends the following guidelines for inventory record accuracy: ± 0.2 percent for A items, ± 1 percent for B items, and ± 5 percent for C items. A items are counted frequently, B items are counted less frequently, and C items are counted the least frequently.

How Much to Order: Economic Order Quantity (EOQ) Models

The question of how much to order is frequently determined by using an **economic order quantity (EOQ)** model. EOQ models identify the optimal order quantity by minimizing the sum of certain annual costs that vary with order size. Three order size models are:

1. The basic economic order quantity model.

2. The economic production quantity model.

3. The quantity discount model.

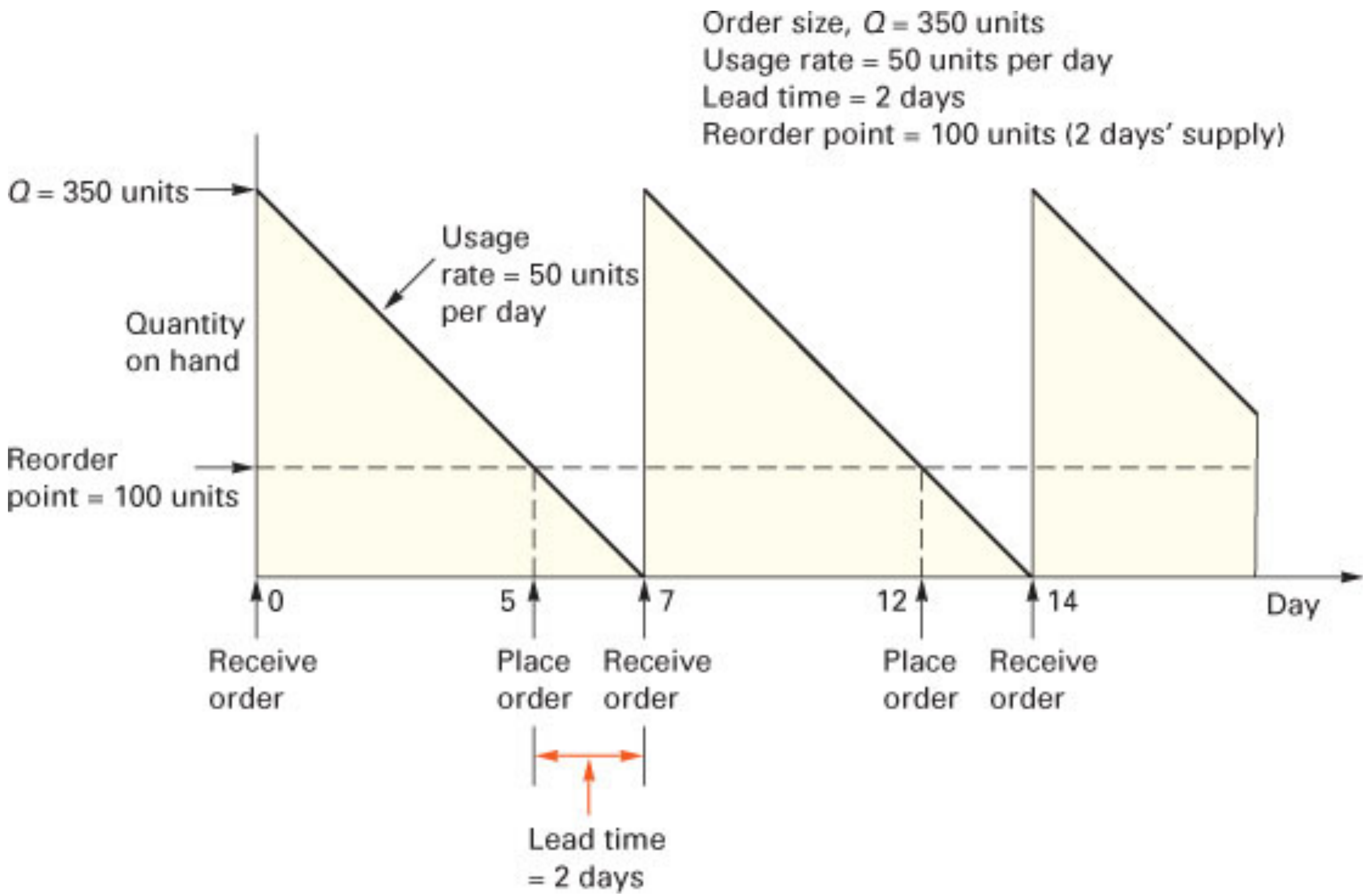
Basic Economic Order Quantity (EOQ) Model

It is used to identify a *fixed* order size that will minimize the sum of the annual costs of holding inventory and ordering inventory.

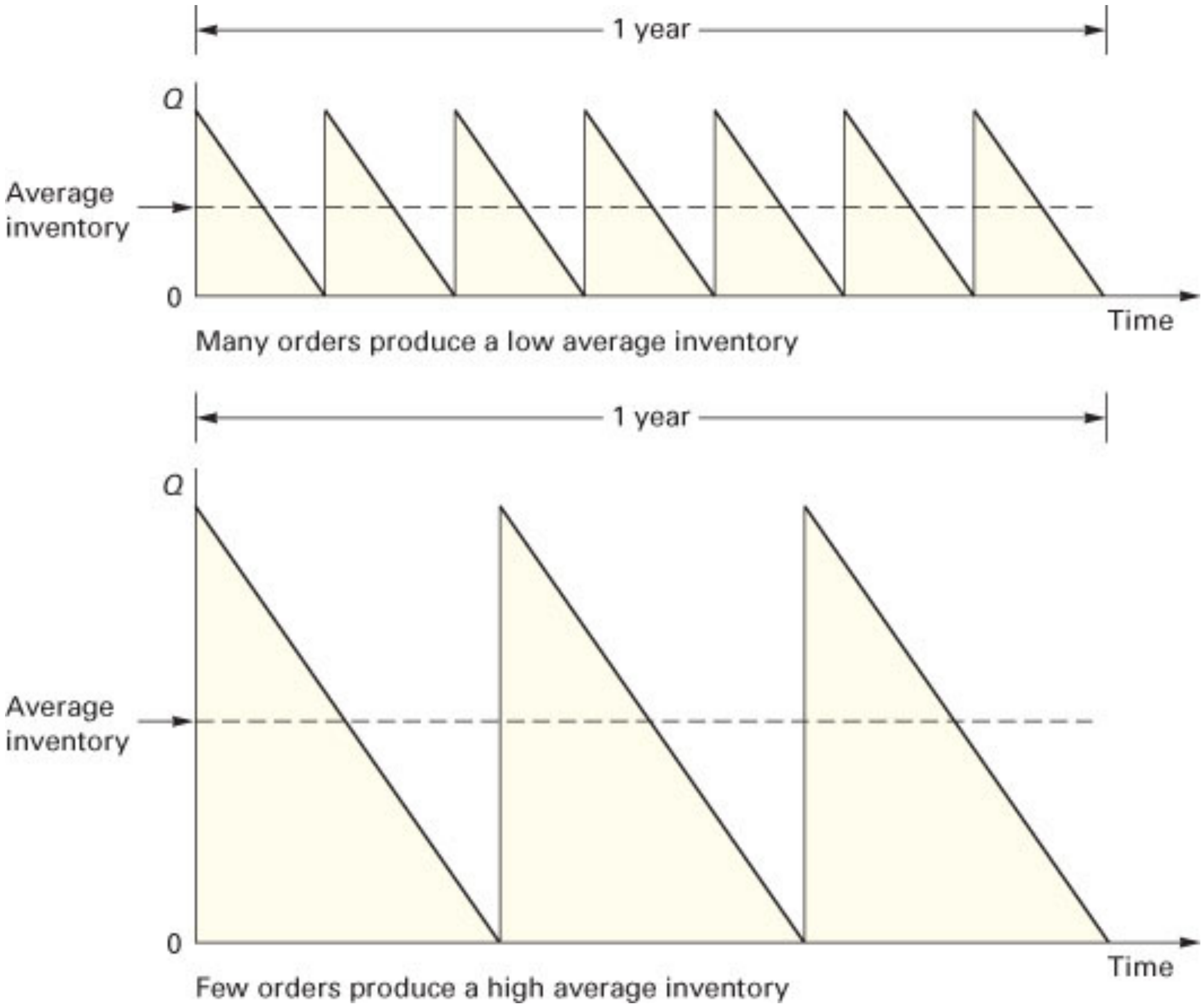
Assumptions of the basic EOQ model:

- Only one product is involved.
- Annual demand requirements are known.
- Demand is spread evenly throughout the year so that the demand rate is reasonably constant.
- Lead time does not vary.
- Each order is received in a single delivery.
- There are no quantity discounts.

The inventory cycle: profile of inventory level over time



Average inventory level and number of orders per year are inversely related: As one increases, the other decreases



The *total annual carrying cost* is

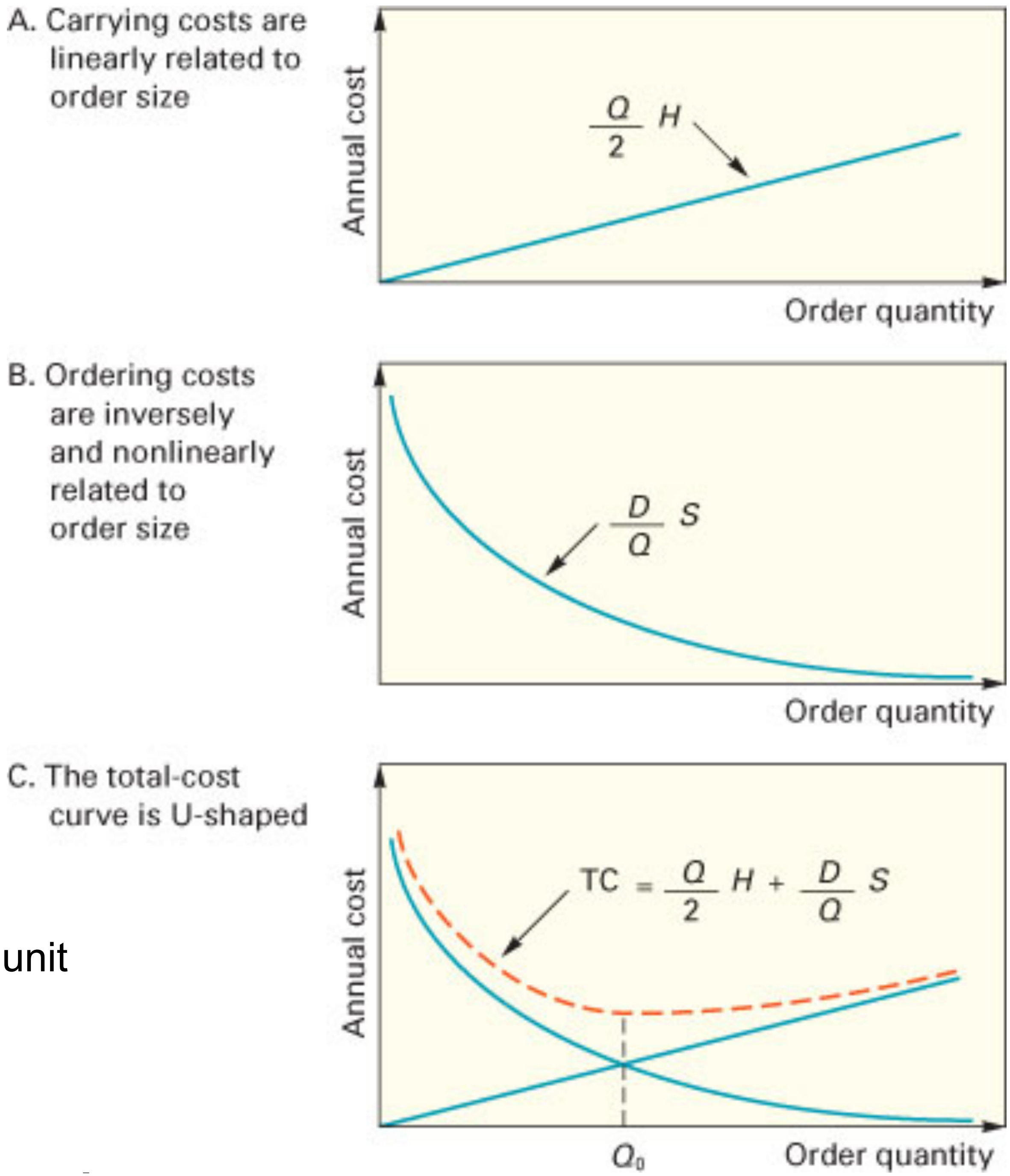
$$\text{Annual carrying cost} = \frac{Q}{2} H$$

where

Q = Order quantity in units

H = Holding (carrying) cost per unit

Carrying cost, ordering cost, and total cost curve



where
D = Annual demand
Q = Order size
H = Holding (carrying) cost per unit
S = Ordering cost
Q_0 = Optimal order quantity

$$\text{Annual ordering cost} = \frac{D}{Q}S$$

$$\text{TC} = \begin{array}{c}\text{Annual}\\ \text{carrying}\\ \text{cost}\end{array} + \begin{array}{c}\text{Annual}\\ \text{ordering}\\ \text{cost}\end{array} = \frac{Q}{2}H + \frac{D}{Q}S$$

$$Q_0 = \sqrt{\frac{2DS}{H}}$$

$$\text{Length of order cycle} = \frac{Q}{D}$$

$$\frac{d\text{TC}}{dQ} = \frac{dQ}{2}H + d(D/Q)S = H/2 - DS/Q^2$$

$$0 = H/2 - DS/Q^2, \text{ so } Q^2 = \frac{2DS}{H} \text{ and } Q = \sqrt{\frac{2DS}{H}}$$

Example

A local distributor for a national tire company expects to sell approximately 9,600 steel-belted radial tires of a certain size and tread design next year. Annual carrying cost is $16 per tire, and ordering cost is $75. The distributor operates 288 days a year.

- What is the EOQ?
- How many times per year does the store reorder?
- What is the length of an order cycle?
- What is the total annual cost if the EOQ quantity is ordered?

D = 9,600 tires per year

H = \$16 per unit per year

S = \$75

a. $Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(9{,}600)75}{16}} = 300 \text{ tires}$

b. Number of orders per year: $D/Q = \frac{9{,}600 \text{ tires}}{300 \text{ tires}} = 32.$

c. Length of order cycle: $Q/D = \frac{300 \text{ tires}}{9{,}600 \text{ tires/yr}} = \frac{1}{32}$ of a year, which is $\frac{1}{32} \times 288$, or nine workdays.

d.

TC = Carrying cost + Ordering cost

= $(Q/2)H + (D/Q)S$

= (300/2)16 + (9,600/300)75

= \$2,400 + \$2,400

= \$4,800

Example

Piddling Manufacturing assembles security monitors. It purchases 3,600 black-and-white cathode ray tubes a year at $65 each. Ordering costs are $31, and annual carrying costs are 20 percent of the purchase price. Compute the optimal quantity and the total annual cost of ordering and carrying the inventory.

D = 3,600 cathode ray tubes per year

S = \$31

H = .20(\$65) = \$13

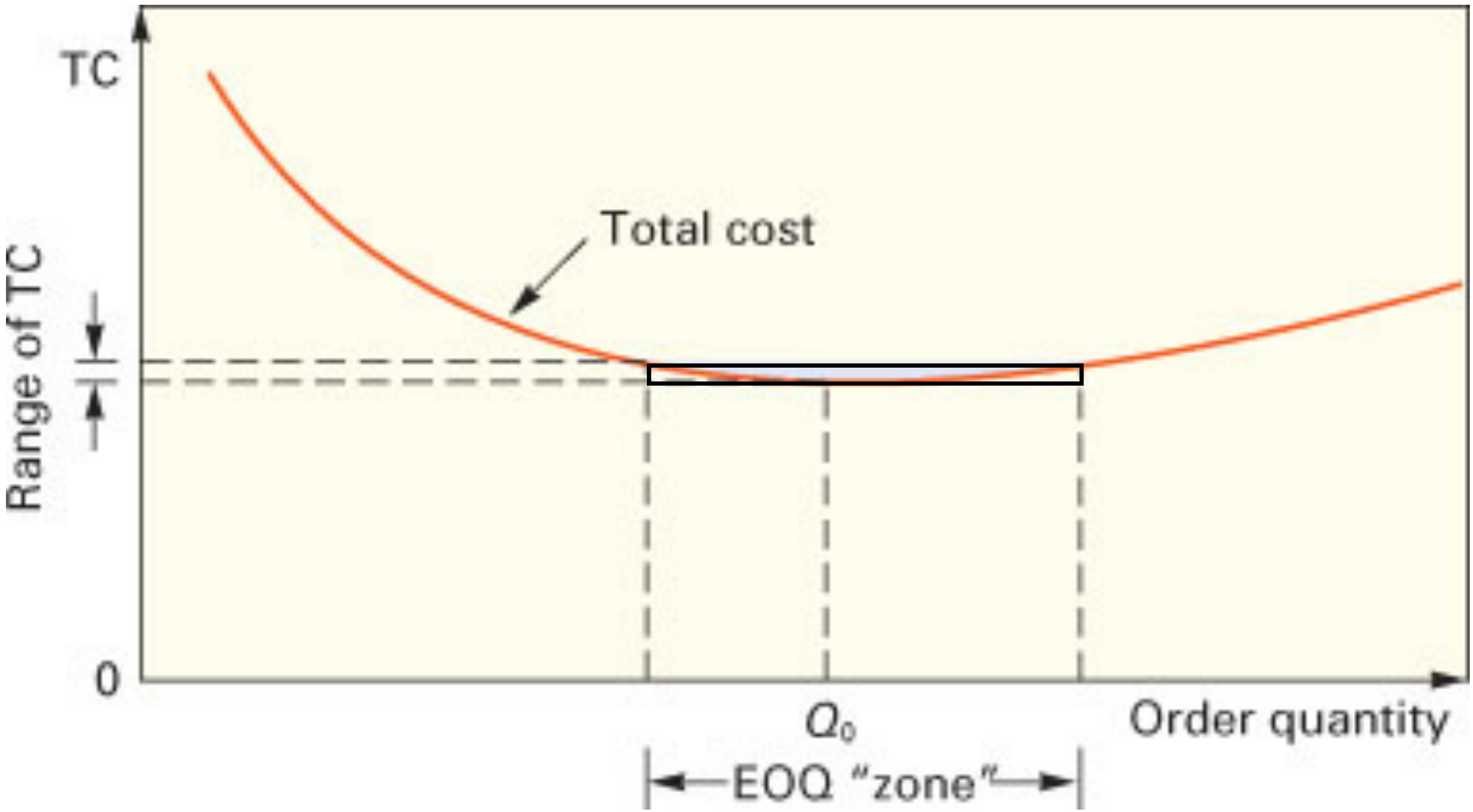
$$Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(3{,}600)(31)}{13}} \approx 131 \text{ cathode ray tubes}$$

TC = Carrying costs + Ordering costs

= $(Q_0/2H) + (D/Q_0)S$

= (131/2)13 + (3,600/131)31

= \$852 + \$852 = \$1,704

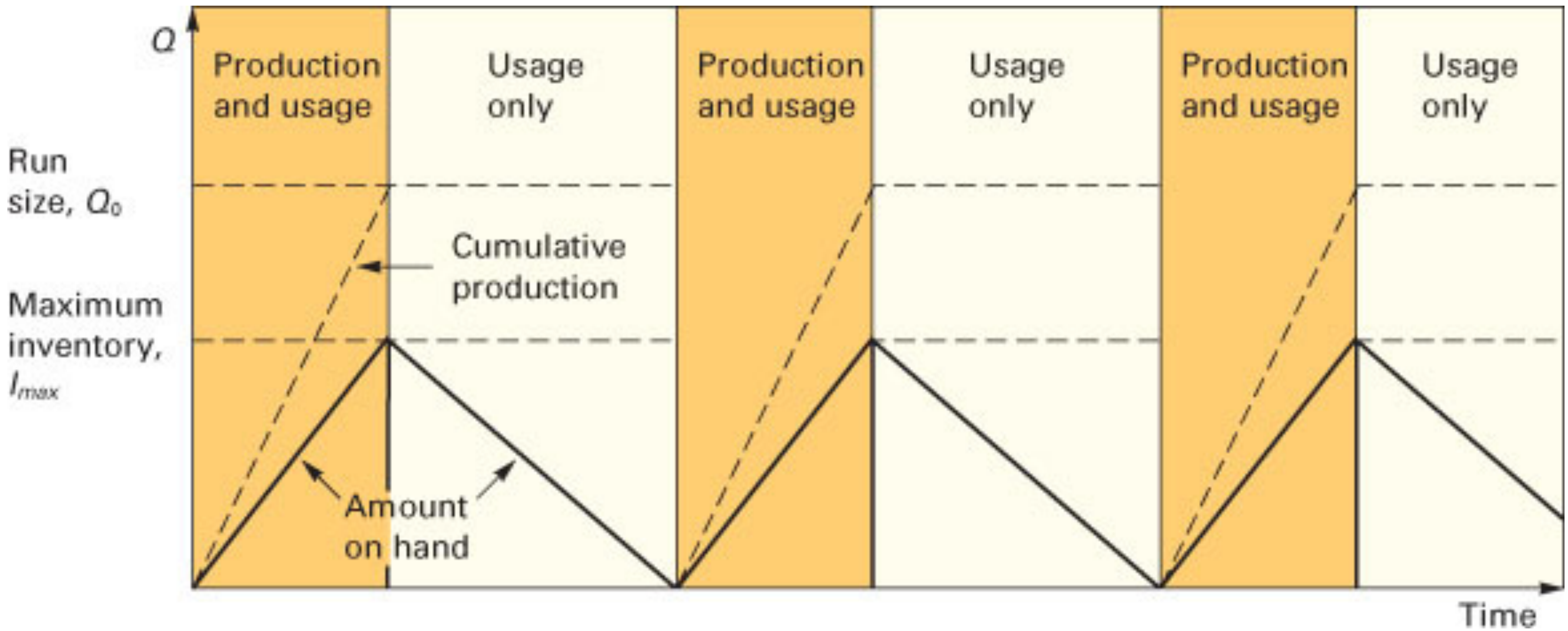

TC
Total cost
Range of TC
0
Q_0
Order quantity
EOQ "zone"

Economic Production Quantity (EPQ)

The assumptions are:

- Only one item is involved.
- Annual demand is known.
- The usage rate is constant.
- Usage occurs continually, but production occurs periodically.
- The production rate is constant.
- Lead time does not vary.
- There are no quantity discounts.

EOQ with incremental inventory replenishment



The total cost is

$$\text{TC}_{\min} = \text{Carrying cost} + \text{Setup cost} = \left(\frac{I_{\max}}{2}\right)H + (D/Q_0)S$$

The economic run quantity is

$$Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}}$$

where

I_{max} = Maximum inventory

p = Production or delivery rate

u = Usage rate

$$\text{Cycle time} = \frac{Q_0}{u}$$

$$I_{\max} = \frac{Q_0}{p}(p - u) \qquad \text{and} \qquad I_{\text{average}} = \frac{I_{\max}}{2}$$

Example

A toy manufacturer uses 48,000 rubber wheels per year for its popular dump truck series. The firm makes its own wheels, which it can produce at a rate of 800 per day. The toy trucks are assembled uniformly over the entire year. Carrying cost is $1 per wheel a year. Setup cost for a production run of wheels is $45. The firm operates 240 days per year. Determine the

- Optimal run size.
- Minimum total annual cost for carrying and setup.
- Cycle time for the optimal run size.
- Run time.

D = 48,000 wheels per year

S = \$45

H = \$1 per wheel per year

p = 800 wheels per day

u = 48,000 wheels per 240 days, or 200 wheels per day

a. $$Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(48{,}000)45}{1}}\sqrt{\frac{800}{800-200}} = 2{,}400 \text{ wheels}$$

b. $$TC_{\min} = \text{Carrying cost} + \text{Setup cost} = \left(\frac{I_{\max}}{2}\right)H + (D/Q_0)S$$

$$I_{\max} = \frac{Q_0}{p}(p-u) = \frac{2{,}400}{800}(800-200) = 1{,}800 \text{ wheels}$$

$$TC = \frac{1{,}800}{2} \times \$1 + \frac{48{,}000}{2{,}400} \times \$45 = \$900 + \$900 = \$1{,}800$$

c. $$\text{Cycle time} = \frac{Q_0}{u} = \frac{2{,}400 \text{ wheels}}{200 \text{ wheels per day}} = 12 \text{ days}$$

d. $$\text{Run time} = \frac{Q_0}{p} = \frac{2{,}400 \text{ wheels}}{800 \text{ wheels per day}} = 3 \text{ days}$$

Quantity Discounts

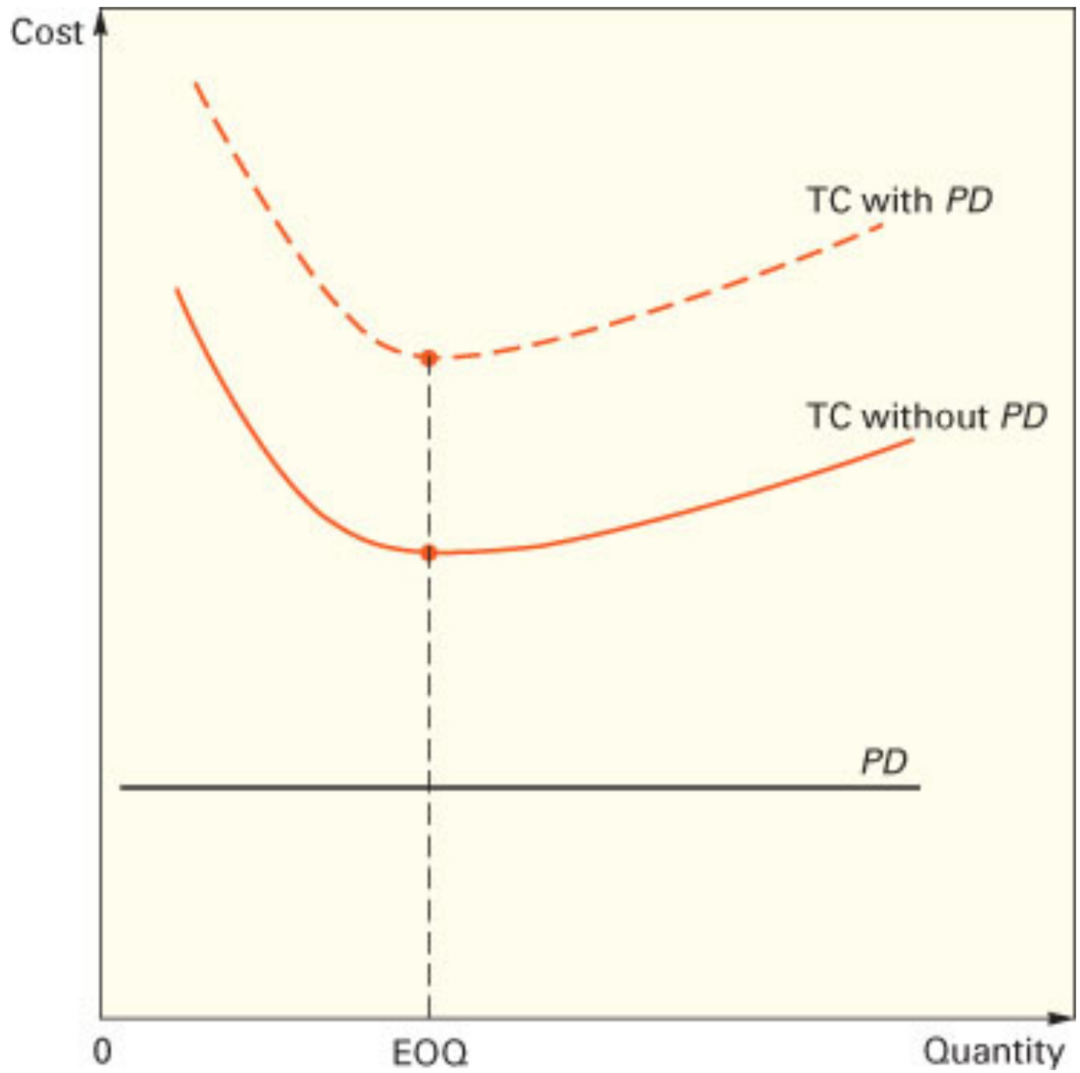
Quantity discounts are price reductions for large orders offered to customers to induce them to buy in large quantities. For example, a Chicago surgical supply company publishes the price list shown in table for boxes of gauze strips. Note that the price per box decreases as order quantity increases.

Order Quantity	Price per Box
1 to 44	$2.00
45 to 69	1.70
70 or more	1.40

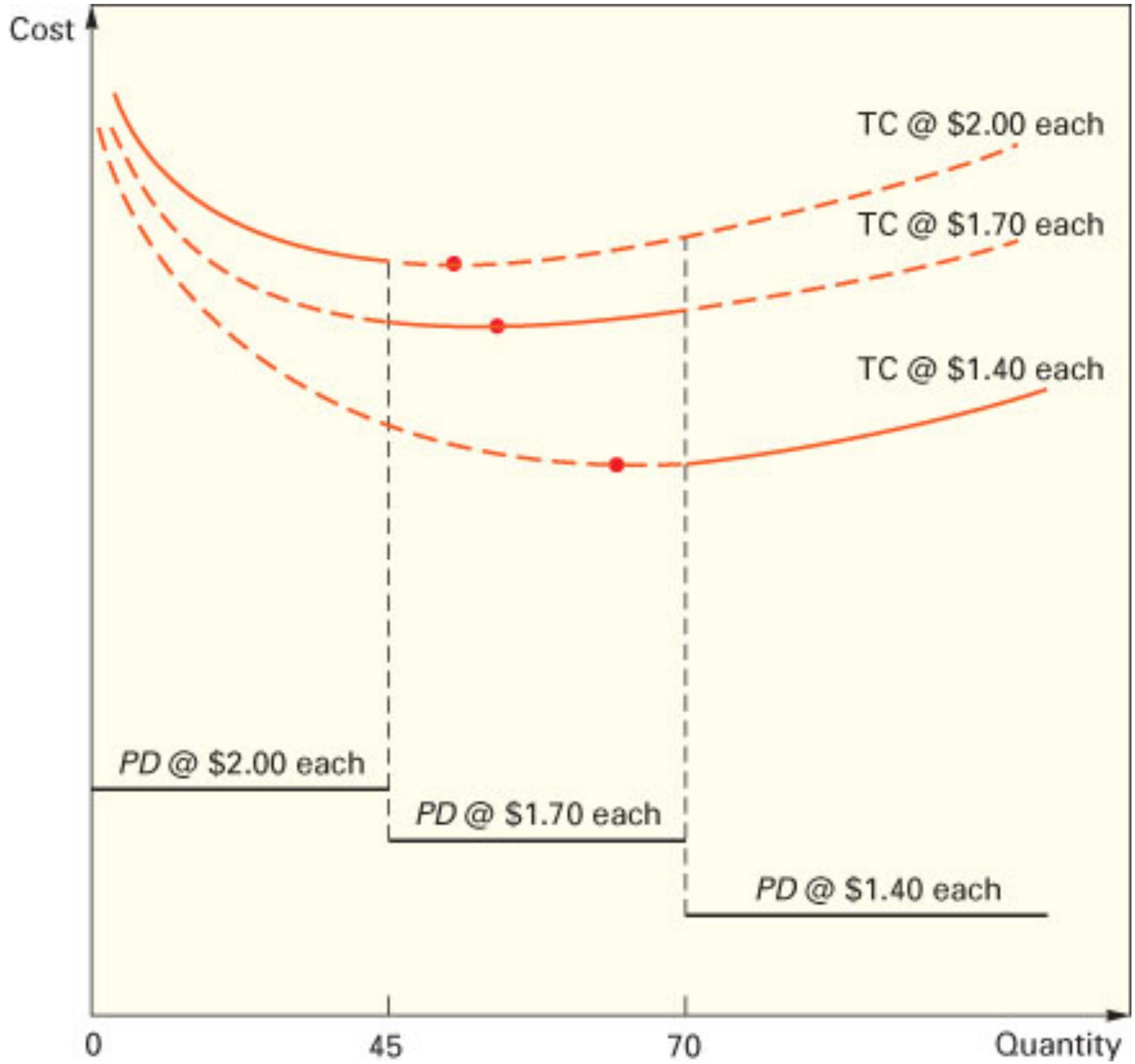
P = Unit price

$$\begin{aligned} \text{TC} &= \text{Carrying cost} + \text{Ordering cost} + \text{Purchasing cost} \\ &= \left(\frac{Q}{2}\right)H + \left(\frac{D}{Q}\right)S + PD \end{aligned}$$

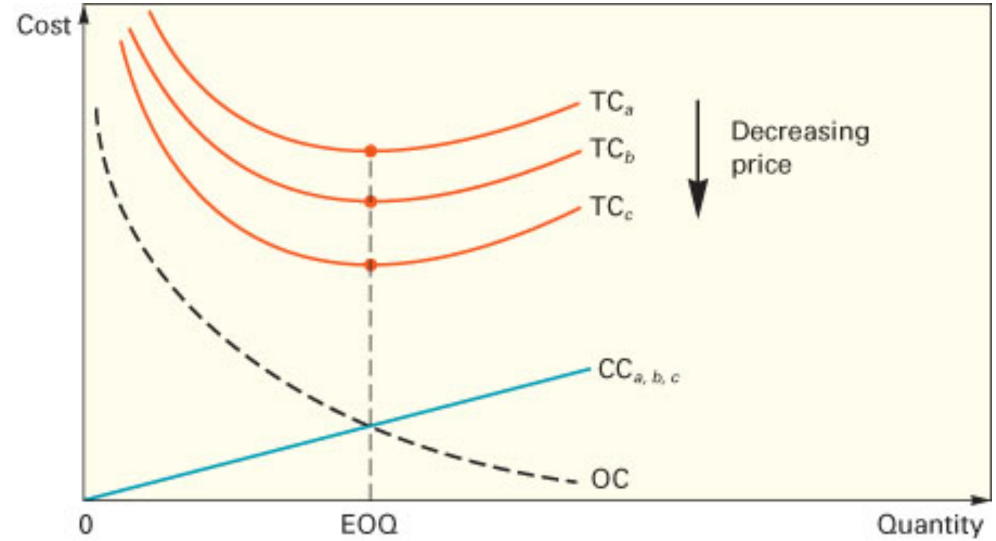
Adding *PD* doesn't change the EOQ



The total-cost curve with quantity discounts is composed of a portion of the total-cost curve for each price

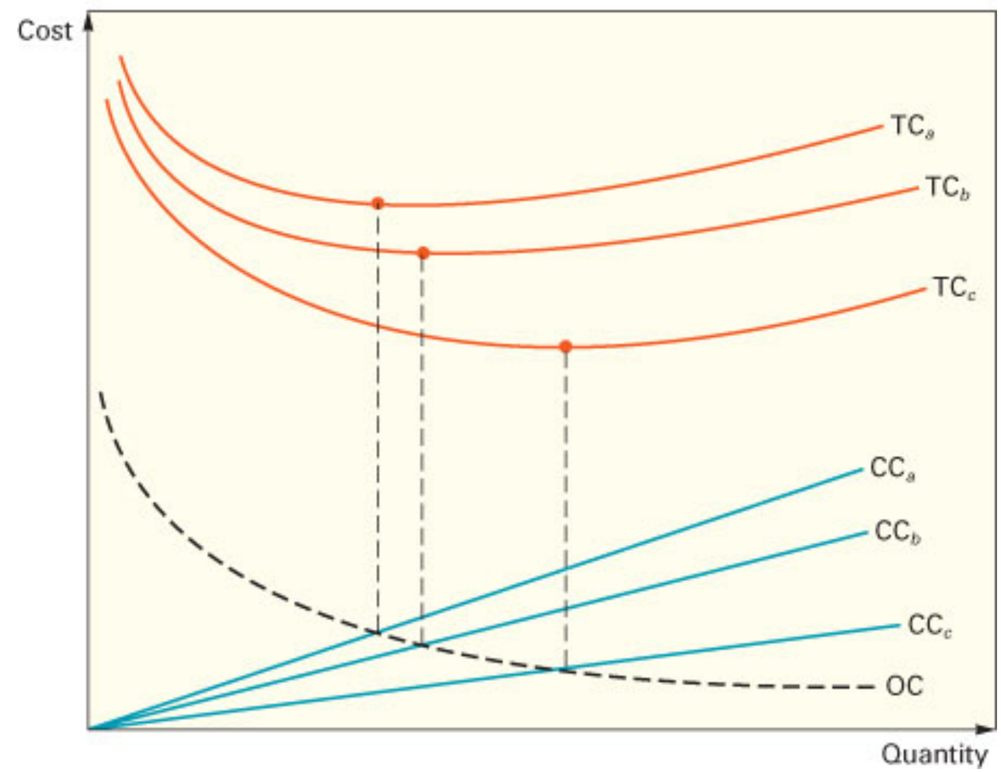


Comparison of TC curves for constant carrying costs and carrying costs that are a percentage of unit costs



A. When carrying costs are constant, all curves have their minimum points at the same quantity.

B. When carrying costs are stated as a percentage of unit price, the minimum points do not line up.



The procedure for determining the overall EOQ differs slightly, depending on which of these two cases is relevant. For carrying costs that are constant, the procedure is as follows:

1. Compute the common minimum point.
2. Only one of the unit prices will have the minimum point in its feasible range since the ranges do not overlap. Identify that range.
   a. If the feasible minimum point is on the lowest price range, that is the optimal order quantity.
   b. If the feasible minimum point is in any other range, compute the total cost for the minimum point and for the price breaks of all *lower* unit costs. Compare the total costs; the quantity (minimum point or price break) that yields the lowest total cost is the optimal order quantity.

Example

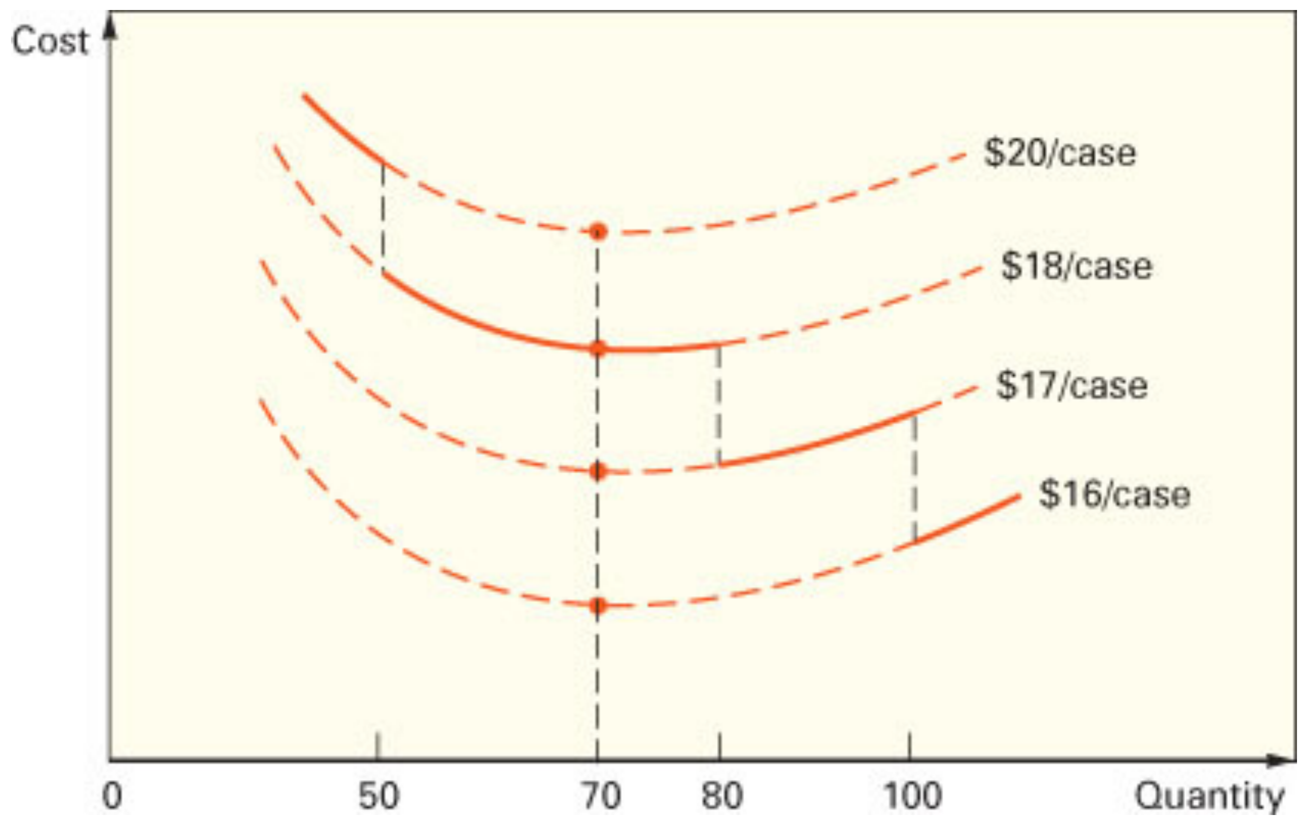
The maintenance department of a large hospital uses about 816 cases of liquid cleanser annually. Ordering costs are \$12, carrying costs are \$4 per case a year, and the new price schedule indicates that orders of less than 50 cases will cost \$20 per case, 50 to 79 cases will cost \$18 per case, 80 to 99 cases will cost \$17 per case, and larger orders will cost \$16 per case. Determine the optimal order quantity and the total cost.

D = 816 cases per year

S = \$12

H = \$4 per case per year

Range	Price
1 to 49	\$20
50 to 79	18
80 to 99	17
100 or more	16



Compute the common EOQ: $= \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(816)12}{4}} = 69.97 \approx 70$ cases

The 70 cases can be bought at \$18 per case because 70 falls in the range of 50 to 79 cases. The total cost to purchase 816 cases a year, at the rate of 70 cases per order, will be

TC = Carrying cost + Order cost + Purchase cost

$TC = (Q/2)H + (D/Q_0)S + PD$

$TC_{70} = (70/2)4 + (816/70)12 + 18(816) = \$14{,}968$

$TC_{80} = (80/2)4 + (816/80)12 + 17(816) = \$14{,}154$

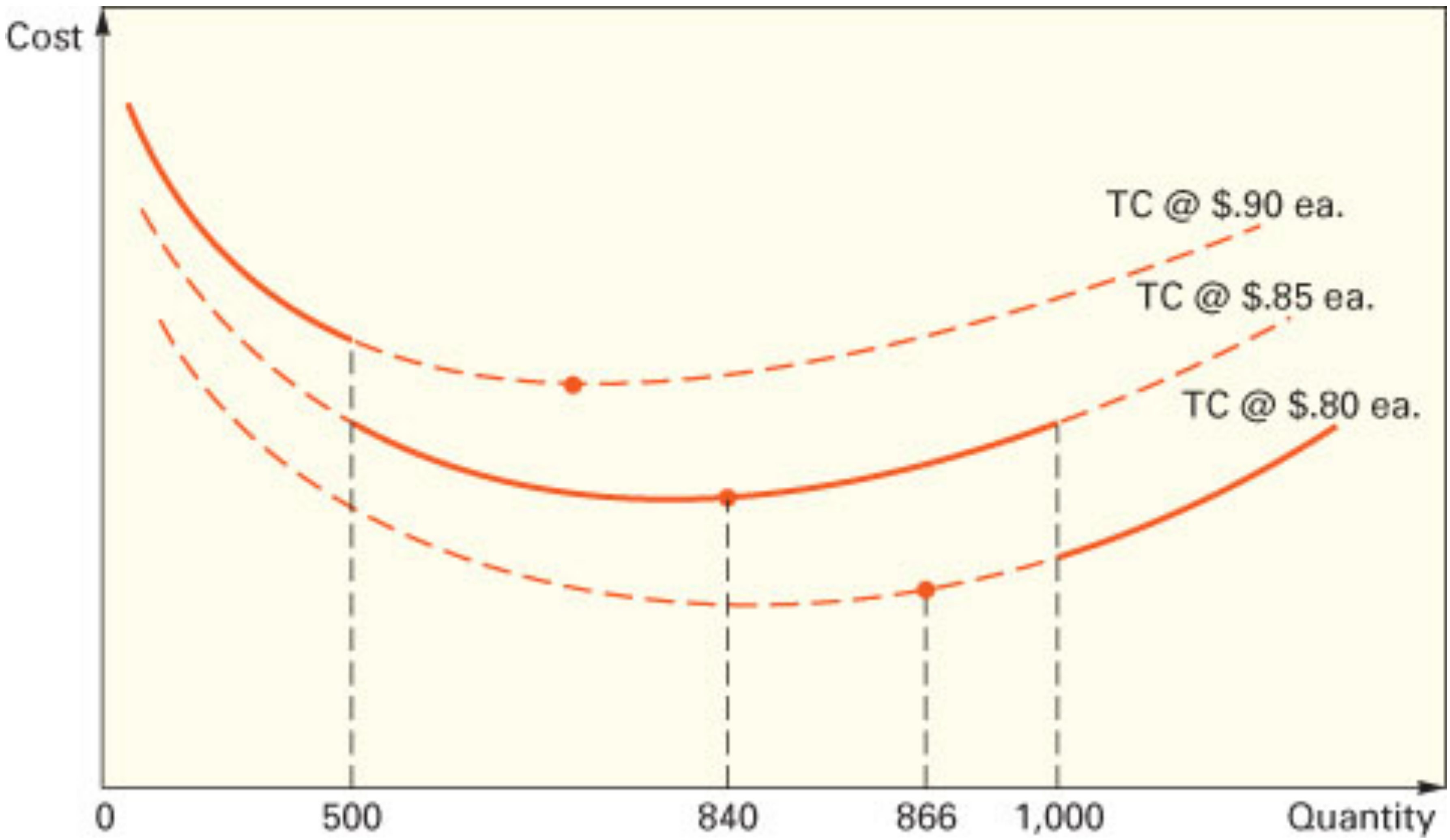
$TC_{100} = (100/2)4 + (816/100)12 + 16(816) = \$13{,}354$

When carrying costs are expressed as a percentage of price, determine the best purchase quantity with the following procedure:

1. Beginning with the lowest unit price, compute the minimum points for each price range until you find a feasible minimum point (i.e., until a minimum point falls in the quantity range for its price).
2. If the minimum point for the lowest unit price is feasible, it is the optimal order quantity. If the minimum point is not feasible in the lowest price range, compare the total cost at the price break for all *lower* prices with the total cost of the feasible minimum point. The quantity that yields the lowest total cost is the optimum.

Example

Surge Electric uses 4,000 toggle switches a year. Switches are priced as follows: 1 to 499, 90 cents each; 500 to 999, 85 cents each; and 1,000 or more, 80 cents each. It costs approximately $30 to prepare an order and receive it, and carrying costs are 40 percent of purchase price per unit on an annual basis. Determine the optimal order quantity and the total annual cost.


Cost
TC @ $.90 ea.
TC @ $.85 ea.
TC @ $.80 ea.
0
500
840
866
1,000
Quantity

D = 4,000 switches per year

S = \$30

H = .40P

Range	Unit Price	H
1 to 499	\$0.90	.40(0.90) = .36
500 to 999	\$0.85	.40(0.85) = .34
1,000 or more	\$0.80	.40(0.80) = .32

$$\text{Minimum point}_{0.80} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(4{,}000)30}{.32}} = 866 \text{ switches}$$

$$\text{Minimum point}_{0.85} = \sqrt{\frac{2(4{,}000)30}{.34}} = 840 \text{ switches}$$

This is feasible; it falls in the $0.85 per switch range of 500 to 999.

Now compute the total cost for 840, and compare it to the total cost of the minimum quantity necessary to obtain a price of $0.80 per switch.

$$TC = \text{Carrying costs} + \text{Ordering costs} + \text{Purchasing costs}$$

$$= \left(\frac{Q}{2}\right)H + \left(\frac{D}{Q}\right)S + PD$$

$$TC_{840} = \frac{840}{2}(.34) + \frac{4{,}000}{840}(30) + 0.85(4{,}000) = \$3{,}686$$

$$TC_{1{,}000} = \frac{1{,}000}{2}(.32) + \frac{4{,}000}{1{,}000}(30) + 0.80(4{,}000) = \$3{,}480$$

Thus, the minimum-cost order size is 1,000 switches.

When to Reorder with EOQ Ordering

There are four determinants of the reorder point quantity:

1. The rate of demand (usually based on a forecast).
2. The lead time.
3. The extent of demand and/or lead time variability.
4. The degree of stockout risk acceptable to management.

$$\text{ROP} = d \times \text{LT}$$

where

d = Demand rate (units per day or week)

LT = Lead time in days or weeks

Note: Demand and lead time must be expressed in the same time units.

Example

Tingly takes Two-a-Day vitamins, which are delivered to his home by a routeman seven days after an order is called in. At what point should Tingly reorder?

Usage = 2 vitamins a day

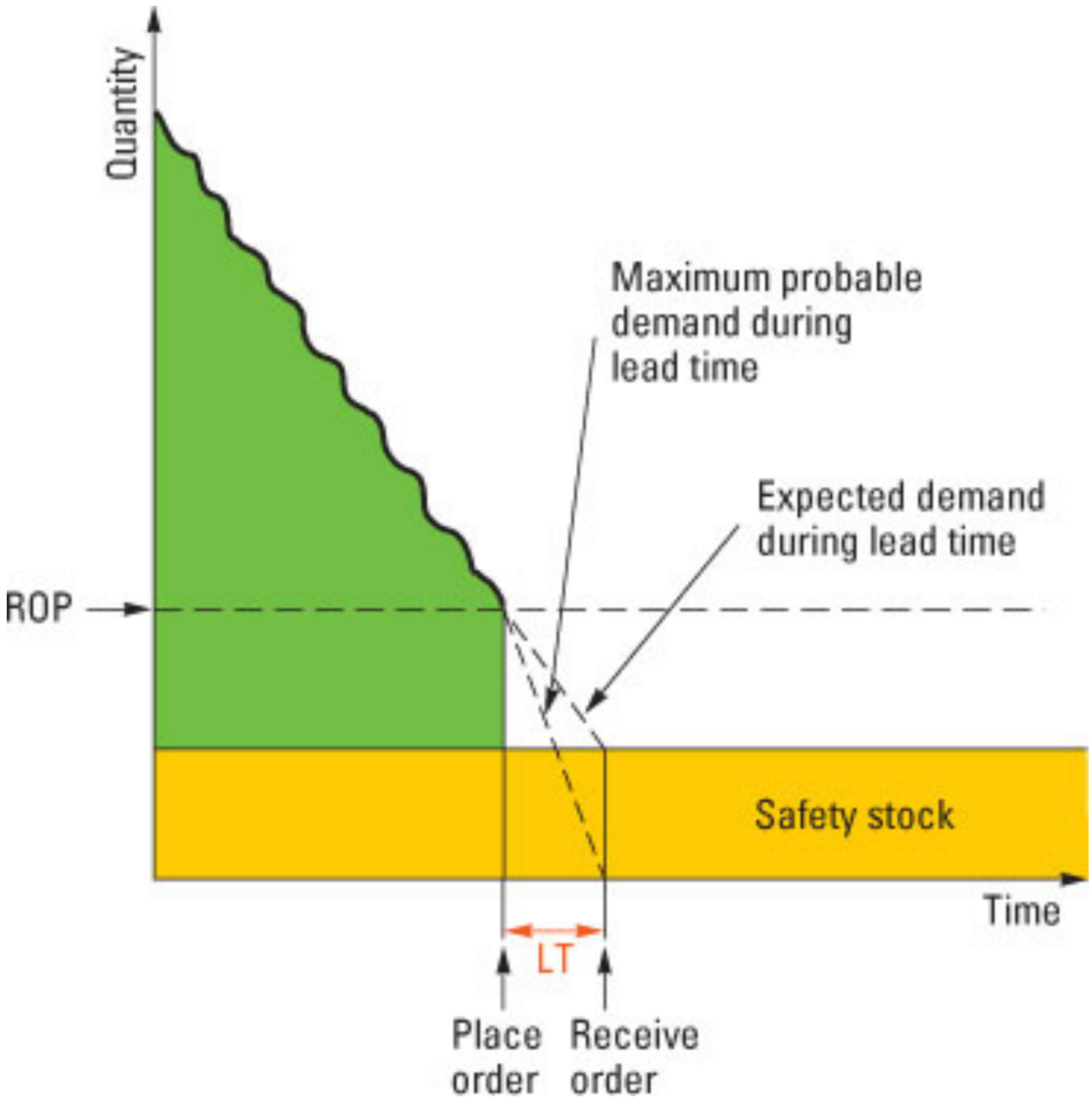
Lead time = 7 days

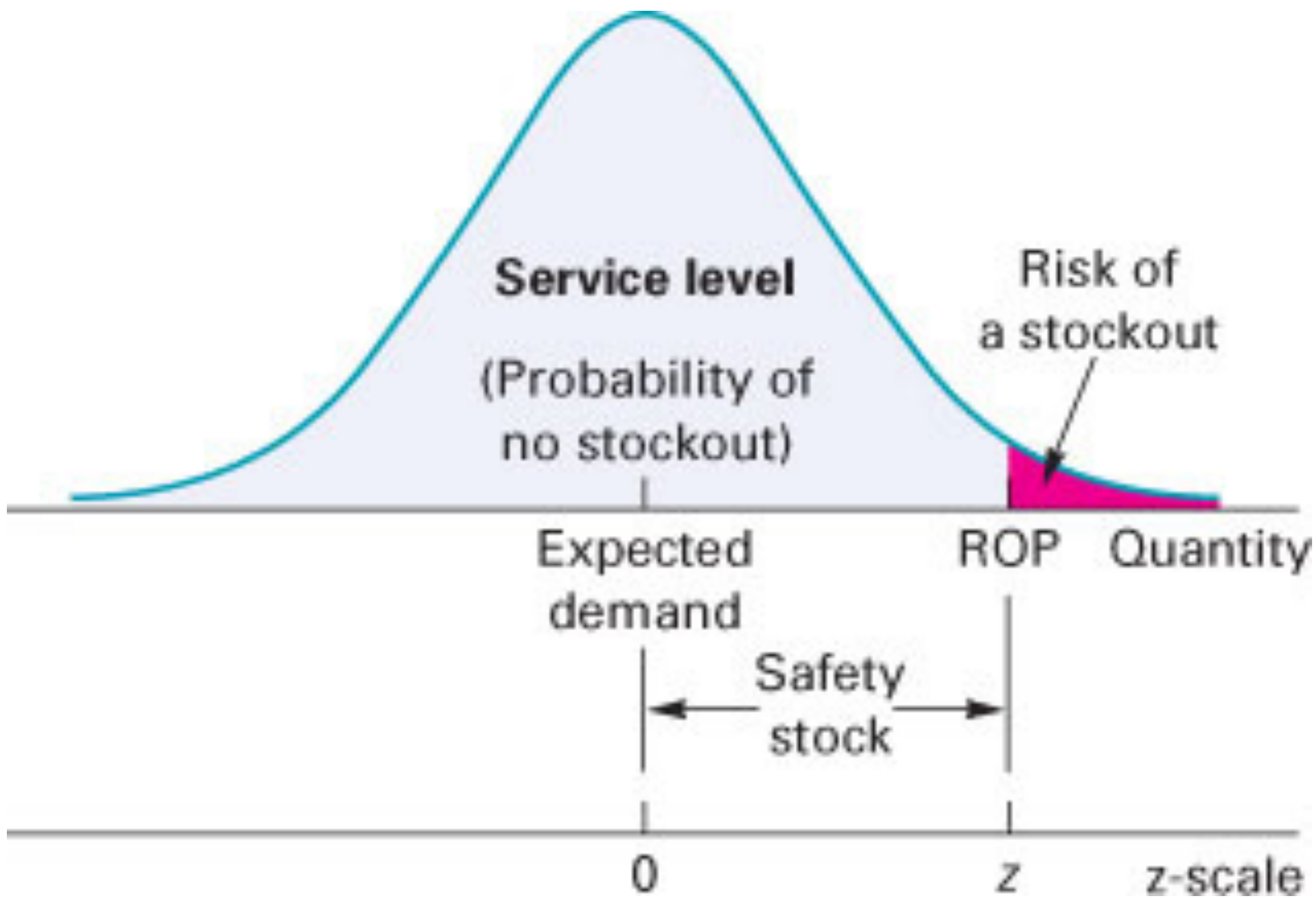
ROP = Usage × Lead time

= 2 vitamins per day × 7 days = 14 vitamins

Thus, Tingly should reorder when 14 vitamin tablets are left.

$$\text{ROP} = \frac{\text{Expected demand}}{\text{during lead time}} + \text{Safety stock}$$


Quantity
Maximum probable demand during lead time
Expected demand during lead time
ROP
Safety stock
Time
LT
Place order
Receive order


Service level
(Probability of
no stockout)
Risk of
a stockout
Expected
demand
ROP
Quantity
Safety
stock
0
z
z-scale

Example

Suppose that the manager of a construction supply house determined from historical records that demand for sand during lead time averages 50 tons. In addition, suppose the manager determined that demand during lead time could be described by a normal distribution that has a mean of 50 tons and a standard deviation of 5 tons. Answer these questions, assuming that the manager is willing to accept a stockout risk of no more than 3 percent:

a. What value of z is appropriate?

b. How much safety stock should be held?

c. What reorder point should be used?

Expected lead time demand = 50 tons

σ_{dLT} = 5 tons

Risk = 3 percent

From Appendix B, Table B, using a service level of 1 – .03 = .9700, you obtain a value of z = +1.88.

Safety stock = $z\sigma_{dLT}$ = 1.88(5) = 9.40 tons

ROP = Expected lead time demand + Safety stock = 50 + 9.40 = 59.40 tons

When data on lead time demand are not readily available, Formula 12-12 cannot be used. Nevertheless, data are generally available on daily or weekly demand, and on the length of lead time. Using those data, a manager can determine whether demand and/or lead time is variable, if variability exists in one or both, and the related standard deviation(s). For those situations, one of the following formulas can be used:

If only **demand** is variable, then , and the reorder point is

$$\sigma_{dLT} = \sigma_d \sqrt{LT}$$

$$ROP = \bar{d} \times LT + z\sigma_d \sqrt{LT}$$

where

= Average daily or weekly demand

c$\bar{d}$ = Standard deviation of demand per day or week

LT = Lead time in days or weeks

If only **lead time** is variable, then $\sigma_{dLT} = d_{\sigma LT}$, and the reorder point is

$$\mathrm{ROP} = d \times \overline{\mathrm{LT}} + zd\sigma_{\mathrm{LT}}$$

where

d = Daily or weekly demand

$\overline{\mathrm{LT}}$= Average lead time in days or weeks

σ_{LT} = Standard deviation of lead time in days or weeks

If both **demand and lead time** are variable, then

$$\sigma_{d\mathrm{LT}} = \sqrt{\overline{\mathrm{LT}}\sigma_d^2 + \bar{d}^2\sigma_{\mathrm{LT}}^2}$$

and the reorder point is $\mathrm{ROP} = \bar{d} \times \overline{\mathrm{LT}} + z\sqrt{\overline{\mathrm{LT}}\sigma_d^2 + \bar{d}^2\sigma_{\mathrm{LT}}^2}$

Example

A restaurant uses an average of 50 jars of a special sauce each week. Weekly usage of sauce has a standard deviation of 3 jars. The manager is willing to accept no more than a 10 percent risk of stockout during lead time, which is two weeks. Assume the distribution of usage is normal.

a. Which of the above formulas is appropriate for this situation? Why?

b. Determine the value of z.

c. Determine the ROP.

a. Because only demand is variable (i.e., has a standard deviation), Formula 12-13 is appropriate.

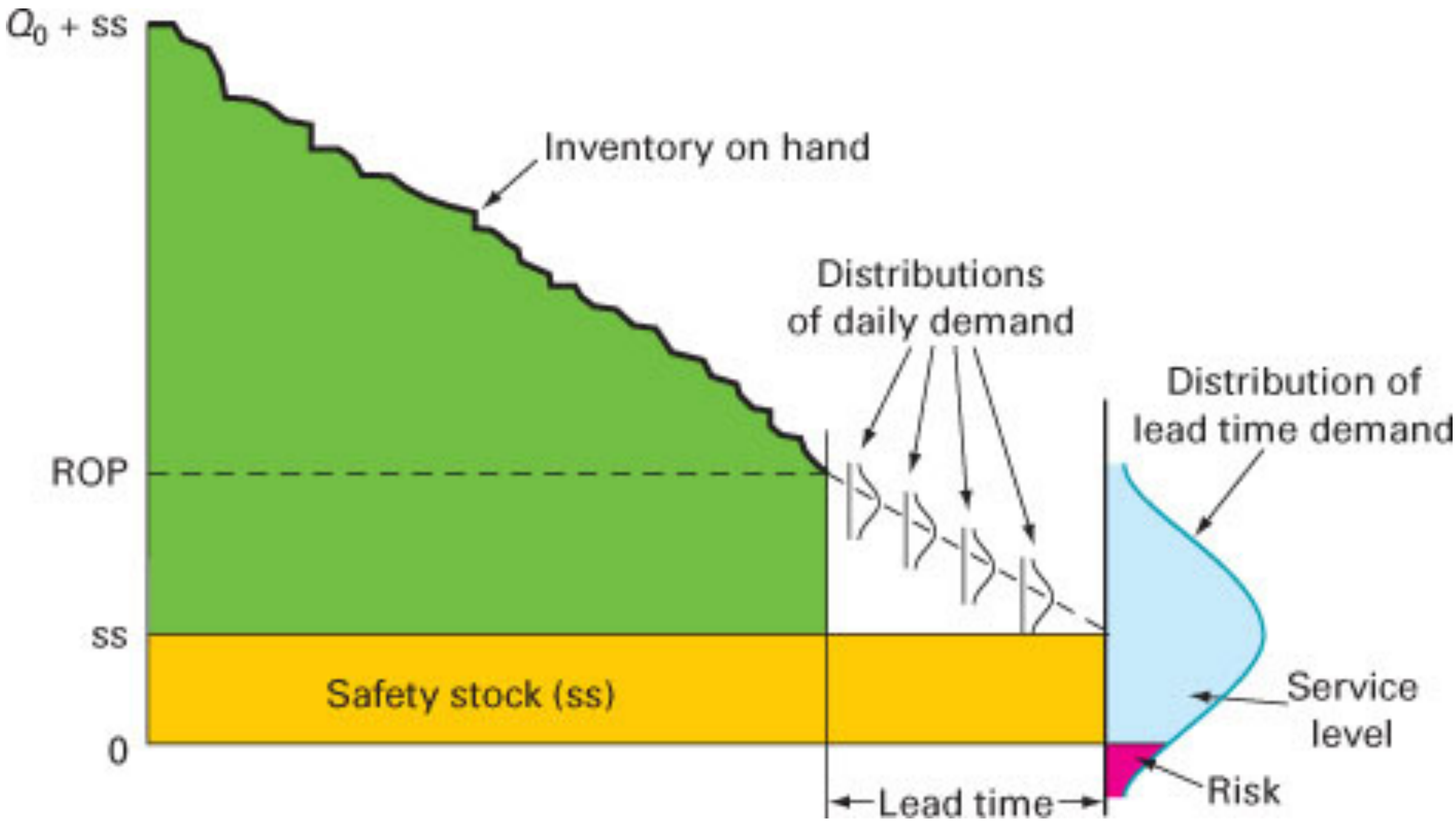
b. From Appendix B, Table B, using a service level of .9000, you obtain $z = +1.28$.

Because the inventory is discrete units (jars), we round this amount to 106. (Generally, round up.)

$\bar{d}$ = 50 jars per week LT = 2 weeks

σ_d = 3 jars per week Acceptable risk = 10 percent, so service level is .90

$$\text{ROP} = \bar{d} \times \text{LT} + z\sigma_d\sqrt{\text{LT}} = 50 \times 2 + 1.28(3)\sqrt{2} = 100 + 5.43 = 105.43.$$



Q_0 + ss
Inventory on hand
Distributions of daily demand
Distribution of lead time demand
ROP
ss
Safety stock (ss)
Service level
0
Risk
Lead time

Shortage and Service Levels

$E(n)$ = Expected number of units short per order cycle

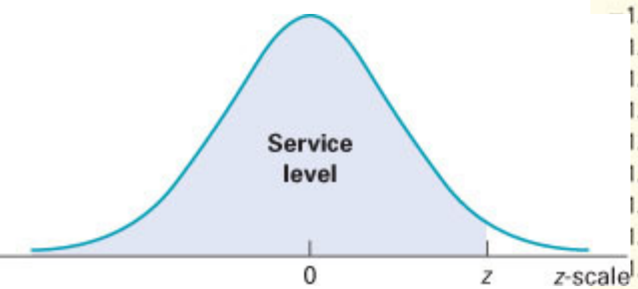
$E(z)$ = Standardized number of units short obtained from table

σ_{dLT} = Standard deviation of lead time demand

$$E(n) = E(z)\sigma_{dLT}$$

Normal distribution service levels and unit normal loss function

z	Lead Time Service Level	E(z)	z	Lead Time Service Level	E(z)	z	Lead Time Service Level	E(z)	z	Lead Time Service Level	E(z)
−2.40	.0082	2.403	−.80	.2119	.920	0.80	.7881	.120	2.40	.9918	.0030
−2.36	.0091	2.363	−.76	.2236	.889	0.84	.7995	.112	2.44	.9927	.0020
−2.32	.0102	2.323	−.72	.2358	.858	0.88	.8106	.104	2.48	.9934	.0020
−2.28	.0113	2.284	−.68	.2483	.828	0.92	.8212	.097	2.52	.9941	.0020
−2.24	.0125	2.244	−.64	.2611	.798	0.96	.8315	.089	2.56	.9948	.0020
−2.20	.0139	2.205	−.60	.2743	.769	1.00	.8413	.083	2.60	.9953	.0010
−2.16	.0154	2.165	−.56	.2877	.740	1.04	.8508	.077	2.64	.9959	.0010
−2.12	.0170	2.126	−.52	.3015	.712	1.08	.8599	.071	2.68	.9963	.0010
−2.08	.0188	2.087	−.48	.3156	.684	1.12	.8686	.066	2.72	.9967	.0010
−2.04	.0207	2.048	−.44	.3300	.657	1.16	.8770	.061	2.76	.9971	.0010
−2.00	.0228	2.008	−.40	.3446	.630	1.20	.8849	.056	2.80	.9974	.0008
−1.96	.0250	1.969	−.36	.3594	.597	1.24	.8925	.052	2.84	.9977	.0007
−1.92	.0274	1.930	−.32	.3745	.576	1.28	.8997	.048	2.88	.9980	.0006
−1.88	.0301	1.892	−.28	.3897	.555	1.32	.9066	.044	2.92	.9982	.0005
−1.84	.0329	1.853	−.24	.4052	.530	1.36	.9131	.040	2.96	.9985	.0004
−1.80	.0359	1.814	−.20	.4207	.507	1.40	.9192	.037	3.00	.9987	.0004
−1.76	.0392	1.776	−.16	.4364	.484	1.44	.9251	.034	3.04	.9988	.0003
−1.72	.0427	1.737	−.12	.4522	.462	1.48	.9306	.031	3.08	.9990	.0003
−1.68	.0465	1.699	−.08	.4681	.440	1.52	.9357	.028	3.12	.9991	.0002
−1.64	.0505	1.661	−.04	.4840	.419	1.56	.9406	.026	3.16	.9992	.0002
−1.60	.0548	1.623	.00	.5000	.399	1.60	.9452	.023	3.20	.9993	.0002
−1.56	.0594	1.586	.04	.5160	.379	1.64	.9495	.021	3.24	.9994	.0001
−1.52	.0643	1.548	.08	.5319	.360	1.68	.9535	.019	3.28	.9995	.0001
−1.48	.0694	1.511	.12	.5478	.342	1.72	.9573	.017	3.32	.9995	.0001
1.44	.0749	1.474	.16	.5636	.324	1.76	.9608	.016	3.36	.9996	.0001
1.40	.0808	1.437	.20	.5793	.307	1.80	.9641	.014	3.40	.9997	.0001
1.36	.0869	1.400	.24	.5948	.290	1.84	.9671	.013			
1.32	.0934	1.364	.28	.6103	.275	1.88	.9699	.012			
1.28	.1003	1.328	.32	.6255	.256	1.92	.9726	.010			
1.24	.1075	1.292	.36	.6406	.237	1.96	.9750	.009			
1.20	.1151	1.256	.40	.6554	.230	2.00	.9772	.008			
1.16	.1230	1.221	.44	.6700	.217	2.04	.9793	.008			
1.12	.1314	1.186	.48	.6844	.204	2.08	.9812	.007			
−1.08	.1401	1.151	.52	.6985	.192	2.12	.9830	.006			
−1.04	.1492	1.117	.56	.7123	.180	2.16	.9846	.005			
−1.00	.1587	1.083	.60	.7257	.169	2.20	.9861	.005			
−.96	.1685	1.049	.64	.7389	.158	2.24	.9875	.004			
−.92	.1788	1.017	.68	.7517	.148	2.28	.9887	.004			
−.88	.1894	0.984	.72	.7642	.138	2.32	.9898	.003			
−.84	.2005	0.952	.76	.7764	.129	2.36	.9909	.003			



Example

Suppose the standard deviation of lead time demand is known to be 20 units. Lead time demand is approximately normal.

a. For a lead time service level of 90 percent, determine the expected number of units short for any order cycle.

b. What lead time service level would an expected shortage of 2 units imply?

$$E(N) = E(n)\frac{D}{Q}$$

Example

Given the following information, determine the expected number of units short per year.

D = 1,000

Q = 250

$E(n)$ = 2.5

$$E(N) = E(n)\frac{D}{Q},$$

$$E(N) = 2.5\left(\frac{1{,}000}{250}\right) = 10.0 \text{ units per year}$$

$$\text{SL}_{\text{annual}} = 1 - \frac{E(N)}{D}$$

$$\text{SL}_{\text{annual}} = 1 - \frac{E(z)\sigma_{dLT}}{Q}$$

Example

Given a lead time service level of .90, D = 1,000, Q = 250, and σ_{dLT} = 16, determine (a) the annual service level, and (b) the amount of cycle safety stock that would provide an annual service level of .98. From table, $E(z)$ = .048 for a 90 percent lead time service level.

$SL_{annual} = 1 - .048(16)/250 = .997$

$.98 = 1 - E(z)(16)/250$

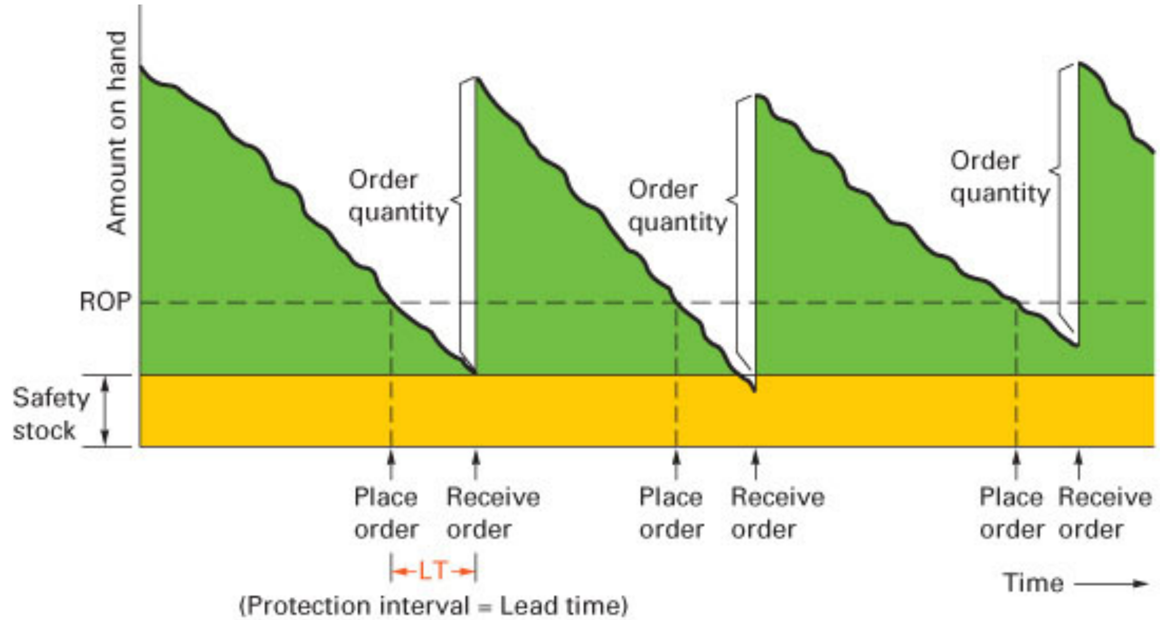
$E(z) = .312$

How Much to Order: Fixed-Order-Interval Model

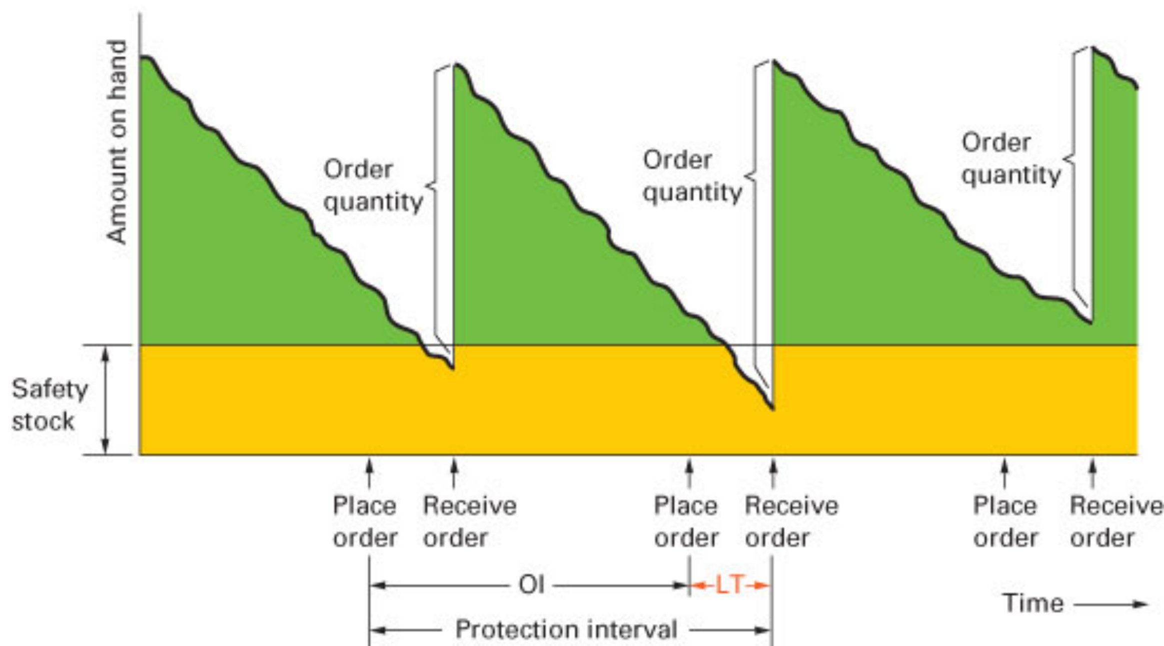
Determining the Amount to Order

Comparison of fixed-quantity and fixed-interval ordering

Fixed quantity



Fixed interval



where

OI = Order interval (length of time between orders)

A = Amount on hand at reorder time

As in previous models, we assume that demand during the protection interval is normally distributed.

$$\begin{aligned}\text{Amount to order} &= \text{Expected demand during protection interval} + \text{Safety stock} - \text{Amount on hand at reorder time} \\ &= \bar{d}(\text{OI} + \text{LT}) + z\sigma_d\sqrt{\text{OI} + \text{LT}} - A\end{aligned}$$

Example

Given the following information, determine the amount to order.

$\bar{d}$ = 30 units per day	Desired service level = 99 percent
σ_d = 3 units per day	Amount on hand at reorder time = 71 units
LT = 2 days	OI = 7 days

$z = 2.33$ for 99 percent service level

$$\begin{aligned}\text{Amount to order} &= \bar{d}(\mathrm{OI} + \mathrm{LT}) + z\sigma_d\sqrt{\mathrm{OI} + \mathrm{LT}} - A \\ &= 30(7 + 2) + 2.33(3)\sqrt{7 + 2} - 71 = 220 \text{ units}\end{aligned}$$

Example

Given the following information:

$$\text{LT} = 4 \text{ days} \qquad A = 43 \text{ units}$$

$$\text{OI} = 12 \text{ days} \qquad Q = 171 \text{ units}$$

$$\bar{d} = 10 \text{ units/day}$$

$$\sigma_d = 2 \text{ units/day}$$

Determine the risk of a stockout at

- The end of the initial lead time.
- The end of the second lead time.

For the risk of stockout for the first lead time, we use Formula 12-13. Substituting the given values, we get

$$43 = 10 \times 4 + z\,(2)(2).$$

Solving, $z = +.75$. From Appendix B, Table B, the service level is .7734. The risk is 1 – .7734 = **.2266**, which is fairly high.

For the risk of a stockout at the end of the second lead time, we use Formula 12-20. Substituting the given values we get

$$171 = 10 \times (4 + 12) + z\,(2)(4) - 43.$$

Solving, $z = +6.75$. This value is way out in the right tail of the normal distribution, making the service level virtually **100** percent, and, thus, the risk of a stockout at this point is essentially equal to zero.

Benefits and Disadvantages

The fixed-interval system results in tight control. In addition, when multiple items come from the same supplier, grouping orders can yield savings in ordering, packing, and shipping costs. Moreover, it may be the only practical approach if inventory withdrawals cannot be closely monitored.

On the negative side, the fixed-interval system necessitates a larger amount of safety stock for a given risk of stockout because of the need to protect against shortages during an entire order interval plus lead time (instead of lead time only), and this increases the carrying cost. Also, there are the costs of the periodic reviews.

The Single-Period Model

$C_{\text{shortage}} = C_s =$ Revenue per unit – Cost per unit

$C_{\text{excess}} = C_e =$ Original cost per unit – Salvage value per unit

Continuous Stocking Levels

The optimal stocking level balances unit shortage and excess costs